FORMULARIO DI MATEMATICA

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Varie

Scomposizioni

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

$(a \pm b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$a^2 - b^2 = (a - b) \cdot (a + b)$

$a^3 \pm b^3 = (a \pm b) \cdot (a^2 \mp ab + b^2)$

$P(x) = a(x - x_1) \cdot (x - x_2) \cdot \ldots \cdot (x - x_n)$

$ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a}$$

Radicali doppi

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}$$

Disequazioni irrazionali

$\sqrt{f(x)} > g(x)$

$$\begin{cases} g(x) \geq 0 \\ f(x) > g^2(x) \end{cases} \cup \begin{cases} g(x) < 0 \\ f(x) \geq 0 \end{cases}$$

$\sqrt{f(x)} < g(x)$

$$\begin{cases} f(x) \geq 0 \\ g(x) > 0 \\ f(x) < g^2(x) \end{cases}$$

Circonferenze inscritte e circoscritte

$R = \frac{abc}{4S}$

$r = \frac{S}{p}$

$r = \frac{c_1 + c_2 - i}{2}$

Logaritmi

$$\log_a 1 = 0$$
$$\log_a a = 1$$
$$\log_a m \cdot n = \log_a m + \log_a n$$
$$\log_a \frac{m}{n} = \log_a m - \log_a n$$
$$\log_a m^{\alpha} = \alpha \cdot \log_a m$$
$$\log_a b = \frac{\log_c b}{\log_c a}$$
$$\log_a b = \frac{1}{\log_b a}$$

Serie aritmetiche

$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{a_1 + a_n}{2} d$$

Serie geometiche

$$a_n = a_1 \cdot q^{n-1}$$
$$S_n = \left\langle \begin{array}{l} a_1 \frac{1-q^n}{1-q}, q \neq 1 \\ na_1, q = 1 \end{array} \right.$$

Combinatoria

Disposizioni **(natura, ordine)**

$$D_{n,k} = \frac{n!}{(n-k)!}$$
$$D'_{n,k} = n^k$$

Combinazioni **(natura)**

$$C_{n,k} = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$
$$C'_{n,k} = \binom{n+k-1}{k}$$

Permutazioni **(ordine)**

$$P_n = n!$$
$$P_n^{k_1, k_2, .., k_{n1}} = \frac{n!}{k_1! \cdot k_2! \cdot \ldots \cdot k_n!}$$

Proprietà

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Probabilità

Evento

- *Certo*: p=1, $E = \Omega$
- *Impossibile*: p=0, $E = \emptyset$
- *Aleatorio*: 0<p<1

- *Semplice*: quando non può essere scomposto ulteriormente, cioè coincide con un punto dello spazio Ω;
- *Composto*: quando è dato dal raggruppamento di eventi semplici.

Dati due eventi A e B, essi si dicono

- *incompatibili* o *disgiunti*: $A \cap B = \emptyset$
- *compatibili*: $A \cap B \neq \emptyset$
- *complementari*: $A \cap B = \emptyset$, $A \cup B = \Omega$

Definizione classica di Laplace

$p = \dfrac{f}{n}$ a condizione che tutti i casi siano equiprobabili

Proprietà fondamentali

$$p(\emptyset) = 0$$

$$p(\Omega) = 1$$

$$0 \leq f \leq n \rightarrow 0 \leq \frac{f}{n} \leq 1 \rightarrow 0 \leq p \leq 1$$

$$p(A^c) = p(\overline{A}) = 1 - p(A)$$

Definizione frequentista (Legge dei Grandi Numeri o Legge Empirica del Caso)

$$p(A) = \lim_{n \to \infty} \frac{f}{n}$$

Legge della somma per eventi incompatibili

$$A \cap B = \emptyset \rightarrow p(A \cup B) = p(A) + p(B).$$

Legge della somma per eventi compatibili

$$A \cap B \neq \emptyset \rightarrow p(A \cup B) = p(A) + p(B) - p(A \cap B).$$

Probabilità Condizionata

$$p(A \setminus B) = \frac{p(A \cap B)}{p(B)}$$

Legge del prodotto per eventi indipendenti (probabilità composta)

A e B indipendenti $\rightarrow p(A \cap B) = p(A) \cdot p(B)$.

Legge del prodotto per eventi non indipendenti

A e B non indipendenti $\rightarrow p(A \cap B) = p(A) \cdot p(B \setminus A)$.

Formula di Bayes

$$p(H_i \setminus E) = \frac{p(H_i) \cdot p(E \setminus H_i)}{\sum_1^n p(H_i) \cdot p(E \setminus H_i)},$$

Distribuzione binomiale o di Bernoulli

$$p_{n,k} = \binom{n}{k} p^k q^{n-k}.$$

Speranza matematica o valor medio

$$M(X) = \sum_1^n x_i p_i .$$

Goniometria

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x} \ \forall x \neq \frac{\pi}{2} + k\pi$$

$$\cot x = \frac{\cos x}{\sin x} \ \forall x \neq k\pi$$

$$\cot x = \frac{1}{\tan x} \ \forall x \neq k\frac{\pi}{2}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\left.\begin{array}{l} \sin\alpha = \dfrac{2t}{1+t^2} \\ \cos\alpha = \dfrac{1-t^2}{1+t^2} \\ \tan\alpha = \dfrac{2t}{1-t^2} \end{array}\right\} t = \tan\frac{\alpha}{2}$$

$$\sin\alpha = \frac{\pm\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$

$$\cos\alpha = \pm\frac{1}{\sqrt{1+\tan^2\alpha}}$$

Formule di Postaferesi

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2}$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Formule di Werner

$$\cos a \sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$

$$\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

$$\cos a \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

Angoli Noti

Rad	*Deg*	*Sin*	*Cos*	*Tg*	*Ctg*
0	0°	0	1	0	**non esiste**
$\frac{\pi}{12}$	15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{8}$	22°30'	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{3}{8}\pi$	67°30'	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
$\frac{5}{12}\pi$	75°	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	**non esiste**	0

Archi associati

Rad	***Sin***	***Cos***	***Tg***	***Ctg***
x	$\sin x$	$\cos x$	$\tan x$	$\cot x$
$\pi - x$	$\sin x$	$-\cos x$	$-\tan x$	$-\cot x$
$\pi + x$	$-\sin x$	$-\cos x$	$\tan x$	$\cot x$
$-x$	$-\sin x$	$\cos x$	$-\tan x$	$-\cot x$
$2\pi - x$	$-\sin x$	$\cos x$	$-\tan x$	$-\cot x$
$\frac{\pi}{2} - x$	$\cos x$	$\sin x$	$\cot x$	$\tan x$
$\frac{\pi}{2} + x$	$\cos x$	$-\sin x$	$-\cot x$	$-\tan x$
$\frac{3}{2}\pi - x$	$-\cos x$	$-\sin x$	$\cot x$	$\tan x$
$\frac{3}{2}\pi + x$	$-\cos x$	$\sin x$	$-\cot x$	$-\tan x$

Trigonometria

Triangolo qualsiasi

AREA

$$S = \frac{1}{2}ab\sin\gamma = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ac\sin\beta$$

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

CORDE

$$\overline{AB} = 2r\sin\alpha$$

SENI

$$\frac{a}{sen\alpha} = \frac{b}{sen\beta} = \frac{c}{sen\gamma} = 2R = \frac{abc}{2S}$$

PROIEZIONI

$$a = b\cos\gamma + c\cos\beta$$

$$b = a\cos\gamma + c\cos\alpha$$

$$c = a\cos\beta + b\cos\alpha$$

CARNOT / COSENO

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Triangolo rettangolo

$$b = a\sin\beta = a\cos\gamma$$

$$c = a\sin\gamma = a\cos\beta$$

$$b = c\tan\beta = c\cot\gamma$$

$$c = b\tan\gamma = b\cot\beta$$

Geometria analitica

Punto e Retta

$y = mx + n; ax + by + c = 0$

$m = -\frac{a}{b}, n = -\frac{c}{b}$

$P_1(x_1, y_1), P_2(x_2, y_2)$

$r_{P_1} : (y - y_1) = m(x - x_1)$

$r_{// P_1} : y_1 = m_{fisso} x_1 + k$

$r_{P_1 P_2} : \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$r // s \Leftrightarrow m_r = m_s$

$r \perp s \Leftrightarrow m_r = -\frac{1}{m_s}$

$\overline{P_1 P_2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$dist(P_1, r) = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Coniche

Circonferenza

$\gamma : x^2 + y_2 + ax + by + c = 0$

$C(x_0, y_0); r \rightarrow (x - x_0)^2 + (y - y_0)^2 = r^2$

$P(x', y') \in \gamma \Leftrightarrow \overline{PC} = r$

$$\begin{cases} a = -2x_0 \\ b = -2y_0 \\ c = x_0^2 + y_0^2 - r^2 \end{cases} \rightarrow \begin{cases} x_0 = -\dfrac{a}{2} \\ y_0 = -\dfrac{b}{2} \\ r = \sqrt{\dfrac{a^2}{4} + \dfrac{b^2}{4} - c} \end{cases}$$

Parabola

$\mathrm{P} : y = ax^2 + bx + c$

$F(x_0, y_0); d : y = k \left\langle \begin{array}{l} y_0 > k \rightarrow a > 0 \rightarrow \cup \\ y_0 < k \rightarrow a < 0 \rightarrow \cap \end{array} \right.$

$P(x', y') \in \mathrm{P} \Leftrightarrow dist(P, F) = dist(P, d)$

$$\begin{cases} a = \dfrac{1}{2y_0 - 2k} \\ b = \dfrac{x_0}{k - y_0} \\ c = \dfrac{x_0^2 + y_0^2 - k^2}{2y_0 - 2k} \end{cases} \rightarrow \begin{cases} x_0 = -\dfrac{b}{2a} \\ y_0 = \dfrac{1 - \Delta}{4a} \\ k = \dfrac{-1 - \Delta}{4a} \end{cases}$$

$F\left(-\dfrac{b}{2a}, \dfrac{1 - \Delta}{4a}\right)$

$d : y = \dfrac{-1 - \Delta}{4a}$

$V\left(-\dfrac{b}{2a}, -\dfrac{\Delta}{4a}\right)$

$A : x = -\dfrac{b}{2a}$

$(\Delta = b^2 - 4ac)$

$\mathrm{P} : x = ay^2 + by + c$

$F(x_0, y_0); d : y = k \left\langle \begin{array}{l} x_0 > k \rightarrow a > 0 \rightarrow \subset \\ x_0 < k \rightarrow a < 0 \rightarrow \supset \end{array} \right.$

$P(x', y') \in \mathrm{P} \Leftrightarrow dist(P, F) = dist(P, d)$

$$\begin{cases} a = \dfrac{1}{2x_0 - 2k} \\ b = \dfrac{y_0}{k - x_0} \\ c = \dfrac{x_0^2 + y_0^2 - k^2}{2x_0 - 2k} \end{cases} \rightarrow \begin{cases} x_0 = \dfrac{1 - \Delta}{4a} \\ y_0 = -\dfrac{b}{2a} \\ k = \dfrac{-1 - \Delta}{4a} \end{cases}$$

$F\left(\dfrac{1 - \Delta}{4a}, -\dfrac{b}{2a}\right)$

$d : x = \dfrac{-1 - \Delta}{4a}$

$V\left(-\dfrac{\Delta}{4a}, -\dfrac{b}{2a}\right)$

$A : y = -\dfrac{b}{2a}$

$(\Delta = b^2 - 4ac)$

Ellisse

$$E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$F_1(-c,0); F_2(c,0)$

$A(-a,0); B(a,0); C(0,b); D(0,-b)$

$P(x',y') \in E \Leftrightarrow \overline{PF_1} + \overline{PF_2} = 2a$

$b^2 = a^2 - c^2 > 0, a > c$

$b < a$

$$e = \frac{c}{a} < 1$$

$$E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$F_1(0,-c); F_2(0,c)$

$A(-a,0); B(a,0); C(0,b); D(0,-b)$

$P(x',y') \in E \Leftrightarrow \overline{PF_1} + \overline{PF_2} = 2a$

$a^2 = b^2 - c^2 > 0, b > c$

$a < b$

$$e = \frac{c}{b} < 1$$

Iperbole

$$I : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$V_1(a,0); V_2(-a,0)$

$F_1(c,0); F_2(-c,0) \in x$

$P(x',y') \in I \Leftrightarrow \left|\overline{PF_1} - \overline{PF_2}\right| = 2a$

$c = \sqrt{a^2 + b^2}$

$a_1 : y = \frac{b}{a}x$

$a_2 : y = -\frac{b}{a}x$

$$e = \frac{c}{a} > 1$$

$$I : \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$V_1(0,b); V_2(0,-b)$

$F_1(0,c); F_2(0,-c) \in y$

$P(x',y') \in I \Leftrightarrow \left|\overline{PF_1} - \overline{PF_2}\right| = 2a$

$c = \sqrt{a^2 + b^2}$

$a_1 : y = \frac{b}{a}x$

$a_2 : y = -\frac{b}{a}x$

$$e = \frac{c}{b} > 1$$

Iperbole equilatera riferita ai suoi assi di simmetria

$\mathrm{I}: x^2 - y^2 = a^2$

$V_1(a,0); V_2(-a,0)$

$F_1(a\sqrt{2},0); F_2(-a\sqrt{2},0) \in x$

$P(x',y') \in \mathrm{I} \Leftrightarrow \left|\overline{PF_1} - \overline{PF_2}\right| = 2a$

$c = a\sqrt{2}$

$a_1 : y = x$

$a_2 : y = -x$

$e = \sqrt{2}$

$\mathrm{I}: x^2 - y^2 = -a^2$

$V_1(0,a); V_2(0,-a)$

$F_1(0,a\sqrt{2}); F_2(0,-a\sqrt{2}) \in y$

$P(x',y') \in \mathrm{I} \Leftrightarrow \left|\overline{PF_1} - \overline{PF_2}\right| = 2a$

$c = a\sqrt{2}$

$a_1 : y = x$

$a_2 : y = -x$

$e = \sqrt{2}$

Iperbole equilatera riferita ai suoi asintoti

$\mathrm{I}: xy = k$

$$k \begin{cases} V_1(\sqrt{k},\sqrt{k}); V_2(-\sqrt{k},-\sqrt{k}) \\ V_1(\sqrt{|k|},-\sqrt{|k|}); V_2(-\sqrt{|k|},\sqrt{|k|}) \end{cases}$$

$a_1 : x = 0$

$a_2 : y = 0$

Iperbole equilatera traslata

$$\mathrm{I}: y = \frac{ax+b}{cx+d}$$

$$\begin{cases} c \neq 0 \\ ad - bc \neq 0 \end{cases}$$

$$a_1 : x = -\frac{d}{c}$$

$$a_2 : y = \frac{a}{c}$$

$e = \sqrt{2}$

Conica generica

$$C : a_{11}x^2 + a_{22}y^2 + a_{12}xy + a_{13}x + a_{23}y + a_{33} = 0$$

$$A = \begin{pmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{2}a_{13} \\ \frac{1}{2}a_{12} & a_{22} & \frac{1}{2}a_{23} \\ \frac{1}{2}a_{13} & \frac{1}{2}a_{23} & a_{33} \end{pmatrix}$$

$$\overline{A} = \begin{pmatrix} a_{11} & \frac{1}{2}a_{12} \\ \frac{1}{2}a_{12} & a_{22} \end{pmatrix}$$

$$|A| = \left\langle \begin{array}{l} |A| = 0 \rightarrow ConDeg; |\overline{A}| = \left\langle \begin{array}{l} |\overline{A}| > 0 \rightarrow \mathrm{Im} \\ |\overline{A}| = 0 \rightarrow \Re \\ |\overline{A}| < 0 \rightarrow // \end{array} \right. \\ |A| \neq 0 \rightarrow Con; |\overline{A}| = \left\langle \begin{array}{l} |\overline{A}| > 0 \rightarrow \mathrm{E} \\ |\overline{A}| = 0 \rightarrow \mathrm{P} \\ |\overline{A}| < 0 \rightarrow \mathrm{I} \end{array} \right. \end{array} \right.$$

Traslazione

$$P(x, y)_{xOy} \mapsto P(X, Y)_{XO_1Y}, O_1(x_0, y_0)$$

$$\begin{cases} X = x - x_0 \\ Y = y - y_0 \end{cases}, \begin{cases} x = X + x_0 \\ y = Y + y_0 \end{cases}$$

produce i termini di 1° grado

Rotazione

$$P(x, y)_{xOy} \mapsto P(X, Y)_{XOY}, x\hat{O}X \cong y\hat{O}Y \cong \alpha$$

$$\begin{cases} X = x\cos\alpha + y\sin\alpha \\ Y = -x\sin\alpha + y\cos\alpha \end{cases}, \begin{cases} x = X\cos\alpha - Y\sin\alpha \\ y = X\sin\alpha + Y\cos\alpha \end{cases}$$

produce il termine di 2° grado misto

Limiti

Forme indeterminate

$\frac{0}{0}$

$\frac{\infty}{\infty}$

$0 \cdot \infty$

$+\infty - \infty$

0^0

∞^0

1^∞

Limiti notevoli

$$\lim_{x \longrightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \longrightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \longrightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \longrightarrow 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \longrightarrow 0} \frac{\log(x + 1)}{x} = 1$$

$$\lim_{x \longrightarrow 0} \frac{\log(1 + \alpha x)}{x} = \alpha$$

$$\lim_{x \longrightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \longrightarrow +\infty} \frac{u^x}{x^\alpha} = +\infty, u > 1$$

$$\lim_{x \longrightarrow 0^+} x^\alpha \log x = 0, \alpha > 0$$

$$\lim_{n \longrightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

$$\lim_{x \longrightarrow +\infty} \frac{\log x}{x^\alpha} = 0, \alpha > 0$$

$$\lim_{x \longrightarrow +\infty} \frac{x^x}{x^\alpha} = +\infty$$

$$\lim_{x \longrightarrow \infty} \frac{P(n)}{Q(n)} = \begin{cases} +\infty : p > q, \frac{a}{b} > 0 \\ -\infty : p > q, \frac{a}{b} < 0 \\ \frac{a}{b} : p = q \\ 0 : p < q \end{cases}$$

$$P(n) = a_p n^p + a_{p-1} n^{p-1} + \ldots + a_1 n + a_0$$

$$Q(n) = b_q n^q + b_{q-1} n^{q-1} + \ldots + b_1 n + b_0$$

Operazioni sui limiti

$l + \infty = +\infty$

$l - \infty = -\infty$

$+\infty + \infty = +\infty$

$-\infty - \infty = -\infty$

$l \neq 0 \cdot \infty = \infty$

$\infty \cdot \infty = \infty$

$\frac{l}{\infty} = 0$

$\frac{\infty}{l_1 \neq 0} = \infty$

$+\infty^{l_1 > 0} = +\infty$

$+\infty^{l_1 < 0} = 0$

$0 < l < 1^{+\infty} = 0$

$l > 1^{+\infty} = +\infty$

$0 < l < 1^{-\infty} = +\infty$

$l > 1^{-\infty} = 0$

Proprietà dei limiti

$f(x) \leq g(x) \leq h(x), \lim_{x \longrightarrow x_0} f(x) = \lim_{x \longrightarrow x_0} h(x) = l \rightarrow \lim_{x \longrightarrow x_0} g(x) = l$

$$\lim_{x \longrightarrow x_0} f(x) = l, \lim_{x \longrightarrow x_0} g(x) = l_1; l, l_1 \in \Re$$

$$\lim_{x \longrightarrow x_0} [f(x) + g(x)] = l + l_1$$

$$\lim_{x \longrightarrow x_0} f(x) \cdot g(x) = l \cdot l_1$$

$$\lim_{x \longrightarrow x_0} k \cdot f(x) = k \cdot l (k \in \Re)$$

$$\lim_{x \longrightarrow x_0} [\lambda \cdot f(x) + \mu \cdot g(x)] = \lambda \cdot l + \mu \cdot l_1 (\lambda, \mu \in \Re)$$

$$\lim_{x \longrightarrow x_0} [f(x)]^n = l^n (n \in N)$$

$$\lim_{x \longrightarrow x_0} f(x) = l \neq 0 \rightarrow \lim_{x \longrightarrow x_0} \frac{1}{f(x)} = \frac{1}{l}$$

$$\lim_{x \longrightarrow x_0} f(x) = \pm\infty \rightarrow \lim_{x \longrightarrow x_0} \frac{1}{f(x)} = 0$$

$$\lim_{x \longrightarrow x_0} \frac{f(x)}{g(x)} = \frac{l}{l_1} (l_1 \neq 0)$$

$$\lim_{x \longrightarrow x_0} |f(x)| = |l|$$

$$\lim_{x \longrightarrow x_0} \log_a f(x) = \log_a l (a \in \Re_0^+ - \{1\})$$

$$\lim_{x \longrightarrow x_0} a^{f(x)} = a^l (a \in \Re_0^+)$$

$$\lim_{x \longrightarrow x_0} [f(x)]^\alpha = l^\alpha (l > 0, \alpha \in \Re)$$

$$\lim_{x \longrightarrow x_0} f(x) = l > 0, \lim_{x \longrightarrow x_0} g(x) = l_1 \rightarrow \lim_{x \longrightarrow x_0} [f(x)]^{g(x)} = l^{l_1}$$

Teorema di de l’Hôpital

$$\left.\begin{array}{l} \lim_{x \longrightarrow x_0} f(x) = \infty \\ \lim_{x \longrightarrow x_0} g(x) = \infty \end{array}\right] \rightarrow \lim_{x \longrightarrow x_0} \frac{f(x)}{g(x)} \left[= \frac{\infty}{\infty} \right] = \lim_{x \longrightarrow x_0} \frac{f'(x)}{g'(x)}$$

Derivate

Definizione

$$f'(x) = D[f(x)] = \frac{df}{dx} = \lim_{h \longrightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Derivate fondamentali

f(x)	**f'(x)**
k	0
x^n	nx^{n-1}
sen$x$	cos$x$
cos$x$	-sen$x$
tg$x$	$1+tg^2x$ oppure $1/cos^2x$
ctg$x$	$-1-ctg^2x$ oppure $-1/sen^2x$
e^x	e^x
a^x	a^xloga
log$x$	$1/x$
$log_a x$	$log_a e/x$
arcsen$x$	$\frac{1}{\sqrt{1-x^2}}$
arccos$x$	$-\frac{1}{\sqrt{1-x^2}}$
arctg$x$	$\frac{1}{1+x^2}$
arcctg$x$	$-\frac{1}{1+x^2}$

Regole di derivazione

$$y = f(x) \pm g(x) \rightarrow y' = f'(x) \pm g'(x)$$

$$y = f(x) \cdot g(x) \rightarrow y' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$y = \frac{f(x)}{g(x)} \rightarrow y' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$y = f\{g[h(x)]\} \rightarrow y' = f'\{g[h(x)]\} \cdot g'[h(x)] \cdot h'(x)$$

$$y = [f(x)]^{g(x)} \rightarrow y' = [f(x)]^{g(x)} \cdot \left[g'(x) \log f(x) + g(x) \frac{f'(x)}{f(x)} \right]$$

$$y = k \cdot f(x) \rightarrow y' = k \cdot f'(x)$$

Integrali

Definizione

$$\int f(x)dx = F(x)+c \Leftrightarrow F'(x) = f(x)$$

$$\int_a^b f(x)dx = F(a) - F(b)$$ ***(Teorema di Torricelli-Barrow)***.

Proprietà dell'integrale indefinito

$$\int kf(x)dx = k\int f(x)dx$$

$$\int [f_1(x) + f_2(x) + ... + f_n(x)]dx = \int f_1(x)dx + \int f_2(x)dx + ... + \int f_n(x)dx$$

Integrali notevoli fondamentali

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int \sin x \cdot dx = -\cos x + c$$

$$\int \cos x \cdot dx = \sin x + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int (1+\tan^2 x) \cdot dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int e^x \cdot dx = e^x + c$$

$$\int a^x \cdot dx = a^x \log_a e + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int \frac{1}{1+x^2} dx = \arctan x + c$$

Altri integrali notevoli

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + c$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\log\left(x+\sqrt{a^2+x^2}\right) + \frac{x\sqrt{a^2+x^2}}{2} + c$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\int \sqrt{1-x^2}\,dx = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x + c$$

$$\int \frac{1}{(x-a)^n}\,dx = \frac{1}{(1-n)(x-a)^{n+1}} + c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \log\left|x+\sqrt{x^2 \pm a^2}\right| + c$$

$$\int \frac{1}{\sqrt{x^2+px+q}}\,dx = \log\left|x+\frac{p}{2}+\sqrt{\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}}\right| + c$$

$$\int \cos ax \cos bx\,dx = \frac{\sin(a+b)x}{2(a+b)} + \frac{\sin(a-b)x}{2(a-b)} + c$$

$$\int \frac{1}{\sqrt{x^2+px+q}}\,dx = \log\left|x+\frac{p}{2}+\sqrt{\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}}\right| + c$$

$$\int \cos \alpha x \cos \beta x\,dx = \frac{\sin(\alpha+\beta)x}{2(\alpha+\beta)} + \frac{\sin(\alpha-\beta)x}{2(\alpha-\beta)} + c$$

$$I_n = \int \sin^{2n} x\,dx = \frac{1}{2n}\left[-\sin^{2n-1} x\cos x + (2n-1)I_{n-1}\right] : I_0 = x + c; I_1 = -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + c$$

$$I_n = \int \log^n x\,dx = x\log^n x - nI_{n-1} : I_0 = x + c; I_1 = x\log x - x + c$$

$$I_n = \int x^n e^x\,dx = x^n e^x - nI_{n-1} : I_0 = e^x + c; I_1 = xe^x - x + c$$

$$I_{n+1} = \int \frac{1}{(1+x^2)^{n+1}}\,dx = \frac{x}{2n(1+x^2)^n} + \frac{2n-1}{2n}I_{n-1} : I_0 = \arctan x + c; I_1 = \frac{x}{2(1+x^2)} + \frac{1}{2}\arctan x + c$$

Regole di integrazione

$$\int [f(x)]^a f'(x)\,dx = \frac{[f(x)]^{a+1}}{a+1} + c, a \neq -1$$

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int \frac{f'(x)}{\cos^2 f(x)}\,dx = \tan f(x) + c$$

$$\int \frac{f'(x)}{\sin^2 f(x)} dx = -\cot f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int f'(x) a^{f(x)} dx = a^{f(x)} \log_a e + c$$

$$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \arcsin f(x) + c$$

$$\int \frac{f'(x)}{1+[f(x)]^2} dx = \arctan f(x) + c$$

Integrazione di funzioni goniometriche

$$\int \sin^{2k+1} x dx = \int \sin^{2k} x \sin x dx = -\int \left(1-\cos^2 x\right)^k d\cos x$$

$$\int \sin^{2k} x dx = \int \left(\frac{1-\cos 2x}{2}\right)^k dx$$

Integrazione di funzioni razionali

$$\int \frac{P_1(x)}{P_2(x)} dx = \int Q(x) dx + \int \frac{R(x)}{P_2(x)} dx, P_1(x) \cdot Q(x) + R(x), \deg[P_2(x)] = 2, \deg[R(x)] = 0 \vee 1$$

$$\Delta P_2 > 0 \rightarrow \int \frac{A}{ax-\alpha_1} dx + \int \frac{B}{ax-\alpha_2} dx = \frac{A}{a} \log|x-\alpha_1| + \frac{B}{a} \log|x-\alpha_2| + c$$

$$\Delta P_2 = 0 \rightarrow \int \frac{A}{x-\alpha} dx + \int \frac{B}{(x-\alpha)^2} dx = A \log|x-\alpha| - \frac{A\alpha+B}{x-\alpha} + c$$

$$\Delta P_2 < 0 \rightarrow \int \frac{gx+h}{ax^2+bx+c} dx = g\int \frac{x}{ax^2+bx+c} dx + h\int \frac{1}{ax^2+bx+c} dx =$$

$$= gs\int \frac{1}{ax^2+bx+c} d\left(ax^2+bx+c\right) + ht\int \frac{1}{(kx+j)^2+1} dx = gs\log(ax^2+bx+c) + ht\arctan(kx+j) + c$$

Tecniche di integrazione

Integrazione per sostituzione

$$x = g(t) \rightarrow \int \{f[g(t)] \cdot g'(t)\} dt$$

Integrazione per parti

$$\int f'(x) g(x) dx = f(x) g(x) - \int f(x) g'(x) dx$$

Volume di solidi di rotazione

$$V = \pi \cdot \int_a^b [f(x)]^2 dx$$

Studio di funzione

1. Dominio
2. Intersezioni con gli assi
3. Segno
4. Limite (asintoti):
   a. asintoto verticale: $\lim_{x \longrightarrow x_0} f(x) = \infty$;
   b. asintoto orizzontale: $\lim_{x \longrightarrow \infty} f(x) = l$: grado denominatore = grado numeratore;
   c. asintoto obliquo: $\lim_{x \longrightarrow \infty} f(x) = \infty$: grado denominatore = grado numeratore – 1:

$$m = \lim_{x \longrightarrow \infty} \frac{f(x)}{x}$$

$$q = \lim_{x \longrightarrow \infty} \left[f(x) - mx \right]$$

5. Derivata prima (crescenza^{+}/decrescenza^{-}; punti di massimo e minimo relativo^{0})
6. Derivata seconda (concavità verso l'alto^{+}/basso^{-}; punti di flesso^{0})